Analytical Modelling of Bossed Membrane

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Abstract – **In early stage of the design process, modelling is a crucial phase because it allows the designer estimating the device performance before fabrication. Very often, this phase is performed with use of FEM simulator that is usually time consuming. In many cases, especially when simple structures are used, the use of analytical model is more convenient. In this paper we present an analytical model of membrane bending containing a boss in its centre. The model concerns the membrane deflection and stress distribution in case of square and rectangular membranes.**

Keywords – **MEMS, modelling, analytical model, membrane, boss.**

I. Introduction

Membrane is one of the most common structures used in MEMS devices. It is used in variety of pressure sensors, capacitive micromachined ultrasonic transducer (CMUT), micropumps, etc. In all of these applications, the membrane has to be properly designed in order to meet the specific requirements. The main parameter is its stiffness that affects the deflection caused by specific load (pressure, force, voltage, etc.). It can be adjusted by varying the membrane dimensions (length, width and thickness) or using different materials (different Young's modulus). Sometimes, specific properties of the membrane cannot be achieved due to required size of the device and technological process. Therefore, some other techniques that stiffens the membrane are desirable. One of them is placing a thick block on the membrane (boss) that does not deflect due to much larger thickness than membrane.

Nowadays, a simulation step is commonly performed in order to find a design with specified performance. In many cases FEM simulation is used. However, this method requires creating a 3-D model of a structure and solving large number of differential equations (depending on mesh density). In case of optimization process, such simulations have to be performed repeatedly until requirements are met. Moreover, each simulation requires rebuilding of the model and updating of the mesh. Including solving time, this process can last relatively long. Thus, the analytical modelling should be more convenient method in the design phase. Moreover, the membrane is a very simple structure that behaviour is described by one simple equation that can be solved by hand. In optimization loop the calculations should last much shorter providing rather precise results.

In general case, analytical modelling of bossed membrane takes the same procedure as for non-bossed membrane. The difference is in boundary conditions. In case of circular membrane the exact solution for deflection can be simply calculated thanks to no dependence in angle dimension (polar coordinates). In case of square and rectangular shapes no exact solution exists and an approximations function has to be applied. Therefore, boss existence changes the approximation function. However, it is possible to apply the dependence from circular membrane into square and rectangular shapes to avoid numerical calculations. This paper presents the methodology of bossed membrane modelling and its analytical model for various shapes.

II. Bossed membrane

Typically, membranes of three shapes are used: circular, square and rectangular with edges ratio equal to 3. Its thickness is much smaller than the substrate thickness that it is fixed to. Therefore, the membrane can be classified as fixed on the edge due to no deflection of the substrate. The membrane stiffness is defined as follows:

$$k = \frac{P}{w}, \tag{1}$$

where P is an applied load (pressure of various source) and w is a membrane deflection caused by applied load. In general, this stiffness has following dependence:

$$k \sim \frac{Eh^3}{x^4}, \tag{2}$$

where E is a Young's modulus, h is a membrane thickness and x is a XY membrane dimension (radius R for square membrane or width b for square/rectangular membranes). Therefore, there are three ways to increase membrane stiffness: choosing material with higher Young's modulus, increasing membrane thickness or decreasing its length/radius. As we know, membrane dimensions cannot be modified without some constrains. The main factor is technological process that limits the minimal size of the membrane. On the other hand, the thickness depends on the membrane fabrication technique (etching, using SOI wafers, deposition). Moreover, the ratio of the XY dimension to the thickness has to fit in specific range to obtain desired bending properties [1]. In most applications, a linear dependence of deflection on applied load is desired. Taking into consideration specific applications, our constrains become more significant. In case of piezoresistive pressure sensor, the size of piezoresitors determines the membrane size. Membrane has to be large enough in order to guarantee uniform stress distribution in regions where piezoresistors are placed. It is necessary to avoid significant sensitivity loss [2]. On the other hand, linear response of the membrane determines the range of measured pressure [3] and yield stress of the material determines maximal applicable pressure that does not damage the membrane. In capacitive micromachined ultrasonic transceiver (CMUT) the membrane stiffness together with its mass is important for obtaining desired resonance frequency [4]. Moreover, higher stiffness is suggested for higher electromechanical efficiency [5]. In other applications even the volume of sealed cavity under the membrane is very important. In example, in high doses radiation sensor [6] the membrane stiffness has to provide specific membrane deflection with pressure increase within the cavity. All above mentioned issues show that membrane design is not a trivial task and some other techniques has to be applied in order to meet specific requirements. One of these is fabricating a membrane with a boss.

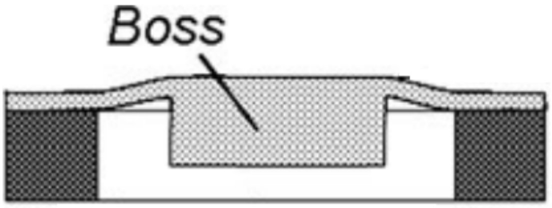

Figure 1: Membrane with a boss

The boss stiffens the membrane [7] that in result may increase the range of linear response of the membrane, increase the range of applicable pressures or shift the resonance frequency of the membrane. The drawback of this technique is the loss in device properties such as sensitivity. Nevertheless, sometimes it is the only one possibility to fabricate working device. Several patterns were developed in order to minimize negative influence [8]. However, the boss in the membrane centre with the same shape as the membrane is commonly used. In most, one boss is enough to obtain desired stiffness and it allows simple analytical modelling.

III. Analytical Modelling of Bossed Membrane

A. Circular membrane

We consider a circular membrane of radius R that is perfectly clamped on its edge. In its centre, there is a circular boss of radius R_b as show in Figure 2. The equation that describes the membrane bending made of isotropic material in polar coordinates for small deflection is [9]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right]\right)=\frac{P}{D}, \tag{3}$$

where w is a membrane deflection, P is an applied load and D is a membrane flexural rigidity described as follows:

$$D=D_0h^3,\ \ D_0=\frac{12E}{(1-\nu^2)}\cdot$$

This equation can be simply integrated and the function of deflection can be obtained:

$$w(r)=\frac{Pr^4}{64D}+\frac{C_1r^2}{4}+C_2\ln\frac{r}{R}+C_3\cdot \tag{4}$$

Following transformation can be applied to obtain dimension-less equation:

$$x=\frac{r}{R},\ \ 0\le x\le 1,\ \ n=\frac{R_b}{R},\ \ 0\le n\le 1\cdot \tag{5}$$

The equation (4) takes the following form:

$$w(x)=\frac{PR^4x^4}{64D}+\frac{C_1R^2x^2}{4}+C_2\ln x+C_3\cdot \tag{6}$$

Applying the boundary conditions:

$$w(1)=0,\ \ \left.\frac{\partial w}{\partial x}\right|_{x=n}=0,\ \ \left.\frac{\partial w}{\partial x}\right|_{x=1}=0 \tag{7}$$

allows calculating constants C_1, C_2 and C_3. The final equation describing membrane deflection is:

$$w(x,n)=\frac{PR^4}{64D}\left[(1-x^2)(1+2n^2-x^2)+4n^2\ln x\right]\cdot \tag{8}$$

Equation (8) describes membrane deflection in any point of the membrane and for any size of the boss. Usually, the maximal deflection is important. Thus, it occurs at the edge of the boss:

$$w_{\max}(n)=w(n,n)=\frac{PR^4}{64D}\left(1-n^4+4n^2\ln n\right)\cdot \tag{9}$$

The above mentioned equation can be also used in case of anisotropic material assuming that anisotropy influences only maximal deflection [10]. In general form, the membrane maximal deflection in function of boss size is as follows:

$$w_{\max}(n)=w_{\max}\left(1-n^4+4n^2\ln n\right), \tag{10}$$

where w_{max} is membrane maximal deflection without boss.

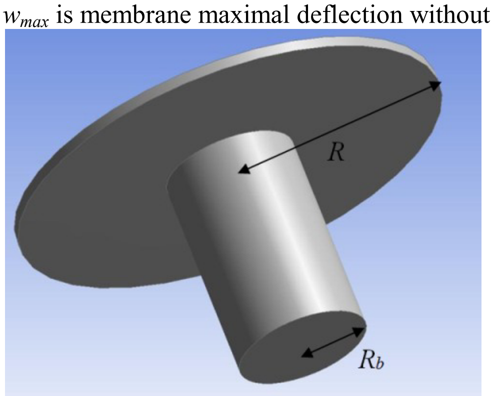

Figure 2: Circular membrane with a boss

B. Rectangular membrane

We consider a rectangular membrane of length a and width b. The equation that describes the membrane bending made of anisotropic material in Cartesian coordinates for small deflection is [9]:

$$\frac{\partial^4 w(x,y)}{\partial x^4}+2\alpha\frac{\partial^4 w(x,y)}{\partial x^2\partial x^y}+\frac{\partial^4 w(x,y)}{\partial y^4}=\frac{P}{D}, \tag{11}$$

where α is a coefficient of anisotropy. In this case there is no possibility to find the solution analytically as it was for circular membrane. The only way is to approximate the solution with a parameterized function and calculate the unknown coefficient (for example [11]). As one can see, the boss on the membrane changes the boundary conditions and therefore, the coefficients in approximation function will be also different. Thus, it is necessary to make calculations in full range of boss radius. To simplify the solution, we will take into consideration only the maximal membrane deflection.

The membrane maximal deflection in analyzed case is [12]:

$$w_{\max} = \frac{1}{C_1}\frac{Pb^4}{D}, \tag{12}$$

where C_1 is a coefficient that depends on ratio a/b and anisotropy coefficient α. The values of this coefficient can be found in [12]. Now, let's take into consideration the membrane with a boss. We assume that the boss has the same shape as the membrane, has the same length a_b to width b_b ratio $a_b/b_b=a/b$ and is placed in the membrane centre (Figure 3).

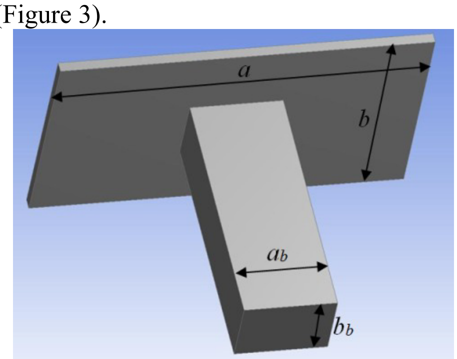

Figure 3: Rectangular membrane with a boss

It can be seen that general equation describing the membrane behaviour has similar form for square and circular membrane. Therefore, we can assume that the dependence of deflection on boss size should be similar as in equation (10). To verify this assumption, simulations were performed in ®ANSYS environment. The values of maximal deflection for boss size in full range and then approximation was performed. The results for square membrane are shown on the Figure 4. As one can see, the results for square membrane perfectly fit the equation (10). It means that the boss changes the membrane stiffness exactly the same way independently on membrane shape. Next, the same procedure was performed for rectangular membrane. Up to ratio a/b equal to 2, the dependence is still the same and equation (10) is valid. It is shown in the Figure 4.

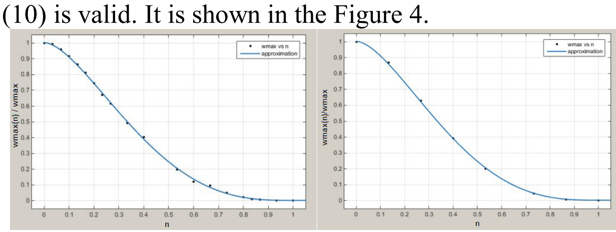

Figure 4: Approximation of deflection for square membrane (left)

Increasing the membrane ratio above 2 causes that the equation (10) is no longer valid due to change of the membrane form. In some range of boss size, the maximal membrane deflection does not occur on the boss edge but on the membrane. It is shown in Figure 5.

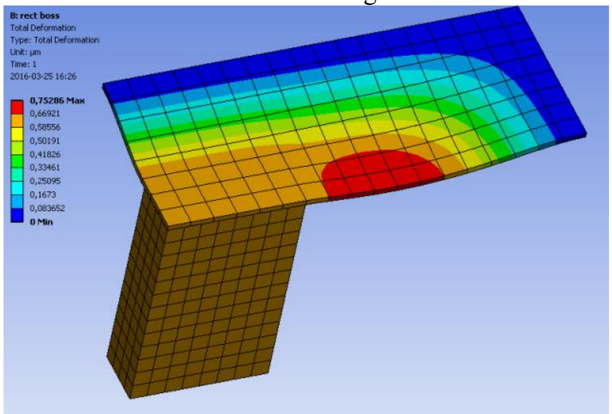

Figure 5: Membrane deflection for rectangular membrane with ratio equal to 2.5 and ratio n equal to 33%

C. *Stress distribution*

Stress induced within the membrane due to its bending depends on membrane convex. Its longitudinal and transverse components on membrane surface are calculated using following formula (in Cartesian coordinates):

$$\sigma_x(x,y) = -\frac{E}{(1-\nu^2)}\left(\frac{\partial^2 w(x,y)}{\partial x^2} + \nu\frac{\partial^2 w(x,y)}{\partial y^2}\right)$$
$$\sigma_y(x,y) = -\frac{E}{(1-\nu^2)}\left(\nu\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right) \tag{13}$$

As one can see, the stress depends on membrane form. Thus, it is not possible to find the exact formula with boss size dependence. Therefore, we will take into consideration only the stress value at most interesting points on the membrane (e.g. in case of piezoresistive pressure sensor): its centre of edges where maximal stress occurs. In case of non-bossed membrane, the stress can be calculated using following formula:

$$\sigma = -\beta\frac{Pb^2}{h^2}, \tag{14}$$

where β is the coefficient that depends on membrane ratio, stress component and point on the membrane and can be found in [9]. The same as for the deflection, the function that approximates the influence of a boss will be taken from circular membrane [13]. The simulations for square membrane showed that the function is correct for one component of the stress (the one whose point of coordinate is not zero). For the other one the function has to be modified. The final formula for stress at the centre of the edge in function of boss size is:

$$\sigma_i(n)\big|_{j=0} = -\beta\frac{Pb^2}{h^2}\left(1-0.9n^2\right)^2,$$
$$\sigma_i(n)\big|_{i=0} = -\beta\frac{Pb^2}{h^2}\left(1-n^2\right), \tag{15}$$

where i and j are Cartesian coordinates (x or y). The results of approximations for square membrane are shown in Figure 6.

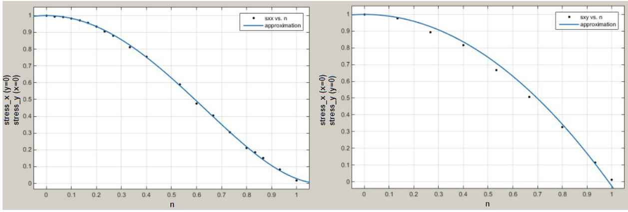

Figure 6: Longitudinal (left) and transverse (right) stresses at centre of one edge of square membrane

In case of rectangular membrane, the same approximation functions can be applied. However, the dependencies for each component of stresses are not the same. The results for rectangular membrane with ratio equal to 2 are presented in Figure 7. Therefore, the dependence may not be valid for other membrane ratios.

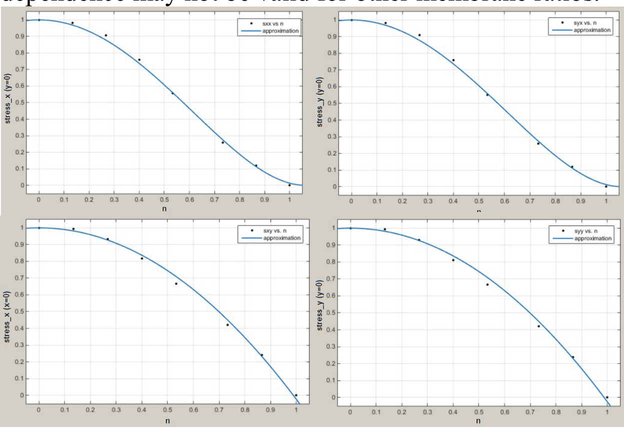

Figure 7: Longitudinal (left) and transverse (right) stresses at centre of longer (up) and shorter edge (down) of rectangular membrane

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IV. CONCLUSIONS

In this paper, the analytical modelling of membrane bending with boss at its centre was described. The analytical solution was performed for circular membrane. In case of square and rectangular membrane it was shown that no analytical solution is possible. Thus, the solution from circular membrane was adopted. The simulations showed good agreement with FEM solution. The limitation of this method is the membrane ratio that should not be greater than 2. Simulations showed that bent membrane has different shape and model is no longer valid. The same procedure was performed for stress distribution. It was shown that analytical formula can be used but it is not valid in whole range of membrane ratio. Thus, the model was presented for square and rectangular membrane with ratio equal to 2. In other cases, the model should be verified or used with much lower precision. Nevertheless, the model gives accurate result and can be used in early stage of the design process and even in optimization but at the end it should be verified with FEM simulation.

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