# An Approach to the Size Reduction for Very Large Circuit Equivalents of MEMS Non-Electrical Parts' Mathematical Models

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Abstract – In this paper the approach to the size reduction for very large circuit equivalents of MEMS non-electrical parts' mathematical models is discussed.

 $\mathit{Keywords}-\mathbf{MEMS}, \mathbf{parallel\ calculations}, \mathbf{Y}\text{-}\Delta\ transformation.$ 

## I. INTRODUCTION

A rapid progress of the modern technologies of microelectromechanical systems (MEMS) production and usage requires a permanent improvement of the computeraided circuit design tools for complex objects with parts of various physical nature. The most widespread approach to solve such tasks is a definition of the object's mathematical model in the flow-difference variable basis, which is a traditional one for electric circuits [1]. However, in that case, the dimensions of circuit equivalents of non-electrical parts' mathematical models, which input data are, as a rule, the finite-difference models defined by the finite element method, could sometimes reach some millions components. This makes it impossible to use even such well-known software packages as SPECTRE [1] or NetALLTED [2] for the circuit design process. The only way out of the situation is a reduction of the initial circuit model dimensions to get a considerably less-sized macromodel but keeping an accuracy required. There are some approaches to solve such tasks [3] but some problems prevent their practical usage taking into account large input data dimensions. This paper is devoted to their solution.

#### **II. PROBLEM STATEMENT**

The one of the most well-known methods to reduce RLC circuit dimensions is the Y- $\Delta$  transformation method. According to the base algorithm [4], when removing *i*-th node connected to nodes *a* and *b* by conductances  $y_a$  and  $y_b$ , a

new conductance  $y_{ab} = (y_a y_b) / Y_i$  where  $Y_i = \sum_{j=1}^k y_j$ , k is a

number of nodes, *i*-th node is connected to, appears and can be presented as

$$y_{ab} = \frac{g_a g_b}{G_i} + p \frac{c_a g_b + c_b g_a}{G_i} + p^2 \frac{4c_a c_b}{G_i}$$
(1)

$$y_{ab} = \frac{1}{p} \frac{b_a b_b}{B_i} + p \frac{c_a b_b + b_a c_b}{B_i} + p^2 (\cdots)$$
(2)

where  $C_i = \sum_{j=1}^{k} c_j$  is a sum of capacitances,  $B_i = \sum_{j=1}^{k} b_j$  is a sum of reactive conductances (reverse inductances),  $G_i = \sum_{j=1}^{k} g_j$  is a sum of conductances connected to node *i*.

The constant coefficient in Eq. (1) is a conductance value that should be inserted between nodes a and b when removing node i, and the p held coefficient is a capacitance value. The l/p held coefficient in Eq. (2) is a reactive conductance value that should be inserted between nodes a and b when removing node i, and the p held coefficient is a capacitance value. Third members of Eqs. (1) and (2) define an Y- $\Delta$  transformation error value. The specific ratios for adding new elements between nodes a and b when removing node i located between them for both cases for all the possible situations are presented in [5].

Having  $Y-\Delta$  transformation used to reduce very large circuit equivalents of MEMS non-electrical parts, some features should be taken into account:

1. For MEMS non-electrical parts' circuit equivalents, R, C, and L component values have almost the same order. This means that an error of Eqs. (1) and (2) could be considerable, especially in the cases when these formulas are using many times. This will influence undoubtedly on a final macromodel's accuracy. So, the Y- $\Delta$  transformation algorithm should be certainly modified to minimize a usage of Eqs. (1) and (2).

2. Having node *i* removed, k(k-1)/2 new elements appear where *k* is a number of nodes next to *i*. This means that if two nodes (with equal other conditions) have different number of connections with next nodes then a number of elements at the circuit to be reduced at the next step could be different also, and larger than at the previous step at that. And this, in turn, increases a usage of Eqs. (1) and (2) to calculate newly created element values.

Let us consider the influence of these factors at the example of finding eigenfrequencies of microaccelerometer's membrane [6].

The microaccelerometer is made as a 50  $\mu$ m square silicon 1  $\mu$ m thickness plate (membrane) 1 (Fig. 1). The sides of the plate lean on a base, and the rest of the surface is free and can bend. There is a metal load 2 in the plate's centre as a 10  $\mu$ m square plate.

When an accelerometer-equipped object moves with an acceleration  $\vec{a}$ , the force of inertia  $\vec{\Phi} = -m\vec{a}$  acts on the load in the opposite direction causing a plate bend. As the respective deformation is an elastic one, there is a dependence between plate's bend and inertia force value, so the object acceleration value could be defined by the bend value.



Fig. 1 Microaccelerometer's calculation scheme.

Calculation of microaccelerometer's eigenfrequencies and mode shapes is executed by a finite element method with ANSYS v10.0 software. For plate's discretization SHELL93  $5x5 \mu m$  square-shaped finite elements are used. The same elements are used to present a central part also, but a coefficient of elasticity and a density are set equal to the respective plate-load material's parameters. Mass and stiffness of the silicon plate under the load are negligible. The microaccelerometer's finite-element model is presented in Fig. 2.



Fig. 2 Microaccelerometer's finite element model.

After getting full-sized arrays describing a status of the system by means of the mor4ansys program, an equivalent electrical circuit was formed in NetALLTED input language, which includes 1883 nodes and 62826 elements. The considered equivalent RLC electrical circuit was reduced by means of the Y- $\Delta$  transformation algorithm to a size of 6 nodes and 30 elements. During the reduction process (Fig. 3), a number of circuit's elements was reaching 735306, a mean number of elements connected to a certain node was reaching 120, and a final error of eigenfrequencies of the macromodel was not great than 10%. Calculations were carried out at the NTUU «KPI»'s supercomputer.

Making an attempt to use SHELL93  $0.5 \times 0.5 \,\mu m$  square elements as finite elements led to the situation that the Y- $\Delta$  transformation algorithm did not finish its work. Exceeding operating memory amount allocated for the work generated an emergency stop event after some working hours. It should be noted that the calculations took place at one of the most powerful nodes of the most powerful in Ukraine NTUU «KPI»'s supercomputer with the following characteristics: two quad-core processors Intel Xeon E5440 with a frequency

of 2.83 GHz and 8 GB of the operating memory. This situation appeared due to a catastrophic splash in a number of newly created elements and data structures related to them during the equivalent circuit reduction process by the base algorithm. This is one of its important disadvantages. This means that it is necessary to give some significant consideration for keeping and finding information in very large data volumes as well as for parallel calculations when very large circuits is being analysed.



Fig. 3 Dependence of a number of circuit components on the reduction step.

#### III. Y- $\Delta$ Transformation Parallel Algorithm

To avoid problems connected with very large input data dimensions, the following Y- $\Delta$  transformation algorithm modification is proposed:

1. Distribute all the input data to n proportionate parts, where n is a number of processors to be used. Allocate the respective data to the each processor's memory.

2. Execute a reduction of the respective electrical circuit by means of the Y- $\Delta$  transformation algorithm on each processor. The interconnection nodes of each subcircuit are considered as non-removable ones.

3. Merge all the separate macromodels obtained in each processor into one task at one processor and repeat the  $Y-\Delta$  transformation algorithm.

To distribute input data effectively, it is reasonable to use a modification of the Sangiovanni-Vincentelli algorithm [7]. It allows reorganizing a circuit's initial structural matrix into the block-diagonal form with bordering. While circuit's nodes appearing in the bordering are non-removable nodes, the diagonal blocks' nodes are respective subcircuits' ones. This Sangiovanni-Vincentelli algorithm modification varies from existing ones in

- changing a block's node inclusion order as it is being formed by a criterion of the minimal increment of its edge;

- developing a new method of the dynamical revaluation of blocks' sizes during decomposition;

- modifying a block size evaluation criterion;

- implementing a statistical search of the optimal block size.

It allows forming a current block with a minimal edge and the maximum possible even block decomposition in whole, thereby minimizing Haydn effect influence and decreasing calculation time due to the even load balance of a multiprocessing system.

To avoid a problem of keeping and finding information in very large data arrays, the following data structures are proposed (Fig. 4).





# IV. NUMERICAL EXPERIMENTS RESULTS

To check a functionality and an efficiency of the suggested approach to solve a reduction task for very large circuit equivalents of MEMS non-electrical parts' mathematical models, a problem of 25  $\mu$ m (0.25x0.25  $\mu$ m covering square elements size) membrane's (Fig. 1) vibration eigenfrequencies calculation is used as one of the tests (Table

1). It should be noted that for this task's initial equivalent circuit, a nodes connectivity k was in the range of 8-105 without taking into account a base node.

A quality of the subcircuit decomposition and a subcircuit quantity *n* influence significantly on a quality of the result obtained and an Y- $\Delta$  transformation parallel algorithm work time. The diagram of the equivalent circuit's node number distribution by subcircuits and non-removable nodes for its

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division by n = 5 is presented in Fig. 5 as one of the best results.

### TABLE 1

MEMBRANE'S MATHEMATICAL MODEL CIRCUIT EQUIVALENT REDUCTION RESULTS

|  | Initial<br>circuit | n=1    | n=5               | n=6              | n=7               |
|--|--------------------|--------|-------------------|------------------|-------------------|
| Non-removable<br>nodes number                          | 2                  | 2      | 3003              | 4004             | 5005              |
| Subcircuits sizes<br>(without non-<br>removable nodes) | _                  | 180801 | 30401-<br>37150   | 29039-<br>30401  | 23031-<br>30401   |
| Execution time, s<br>(days)                            | -                  | -      | 375993<br>(4,35)  | 337726<br>(3,91) | 376126<br>(4,35)  |
| Nodes number   | 180803             | -      | 15                |                  |                   |
| Total elements   | 6880370            | -      | 210               | 210              | 210               |
| Reduct. by nodes, %                                    | -                  | -      | 99,99             | 99,99            | 99,99             |
| Reduct. by elem., %                                    |                    | -      | 99,997            | 99,997           | 99,997            |
| 1 <sup>st</sup> peak, Hz<br>(error, %)                 | 1800,3             | -      | 1778,8<br>(1,19)  | -                | -                 |
| 2 <sup>nd</sup> peak, Hz<br>(error, %)                 | 4676,1             | -      | 4690,3<br>(0,30)  | 5116,6<br>(9,42) | 4278,4<br>(8,50)  |
| 3 <sup>rd</sup> peak, Hz<br>(error, %)                 | 7550,6             | -      | 8544,5<br>(13,16) | 7077,8<br>(6,26) | 6552,6<br>(13,22) |



Fig. 5 The diagram of the equivalent circuit's node number distribution by subcircuits and non-removable nodes for n = 5.

The number of newly created bordering fixed nodes may serve as the main recommendation to select an optimal number n for division on the subcircuits to control accuracy and velocity of the resulting macromodel receiving process: the less their number is, the more accurate results, but if there are large sizes of subcircuits obtained, then this could negatively influence on a result due to the great splashes of a number of newly created elements during reduction. Also it should be taken into account that both an algorithm acceleration coefficient and a number of bordering fixed nodes growth up as a number of subcircuits increases. It could lead to the significant increment of "glued" circuit's connectivity and size that could have a negative influence on the accuracy and total reduction time.

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# V. CONCLUSION

The approach suggested to reduce very large circuit equivalents of MEMS non-electrical parts' mathematical models on a base of the Y- $\Delta$  parallel transformation method allowed reducing significantly a calculation time (tens times for circuits starting with approximately 2000 nodes and 50000 elements) for a test example set. In the same time the accuracy of an obtained macromodel is not worse than the accuracy of a macromodel obtained by means of the base algorithm. Moreover, this approach makes it possible to get macromodels of very large equivalent circuits (for example, 180803 nodes and approximately 6.8 millions elements) what is practically impossible to be done by means of the base algorithm.

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