Monte Carlo Modeling of Stiffness of MEMS Membrane

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***Abstract*—Simulation step is nowadays one of the crucial in the design process of the device. It allows estimating the device performance and reducing the cost of the fabrication process. It is also very useful in statistical approach. As technological process is not ideal and always produce some deviation from established parameters, the final device has variable performance. In this paper, the statistical approach using Monte Carlo modelling is presented in order to estimate the stiffness of a rectangular membrane of various length to width ratio.**

***Keywords*—Monte Carlo; MEMS; membrane; stiffness**

I. Introduction

Membrane structures are the basis of many applications in micromechanics [1]. Wide spectrum of functional targets defines construction of the membrane (in particular – poly layer matrix, composite, profiled, corrugated, perforated, etc.), their geometrical and topological parameters with a range of thickness from 0.1 μm to 100 μm and length/width from a few micrometers to a few millimeters [2]. Because of this, an optimization problem becomes the actual one in order to achieve better performance in sensors.

Monte Carlo analysis is used to investigate the impact of fabrication tolerances on the device performance. Such analysis uses component tolerances and statistical distributions to randomly vary system parameters during successive simulations. This work is deduced to the results of statistical modeling of membranes in terms of theirs stiffness coefficient.

II. Membrane Modelling

The mathematical model of a membrane in a deformed state is described in a form of differential equation [3]

$$D\left(\frac{\partial^4 w_1(x,y)}{\partial x^4}+2\alpha\frac{\partial^4 w_1(x,y)}{\partial x^2\partial y^2}+\frac{\partial^4 w_1(x,y)}{\partial y^4}\right)=P \qquad (1)$$

where $w_1\,(x,y)$ is the normal displacement for a point of the membrane at a location *(x, y)*, *P* is the applied pressure in the direction of and the term *D* represents the rigidity of the membrane, which is related to Young's modulus (*E*), Poisson's ratio (ν) and the thickness of the material (*h*) according to

$$D=\frac{Eh^3}{12(1-\nu^2)} \qquad (2)$$

In the case of a membrane with fixed boundaries, several important qualitative observations can be made:

- The maximum displacement occurs at the center of the membrane.
- The maximum stress occurs at the center points of two opposite edges and in the center of the membrane.
- The stresses along the edge and the center have different signs.
- These locations with high stress values are preferred for the placement of piezoresistive sensors for detecting membrane deformation.

In many applications, only the maximum displacement and the maximum stress are of interest. These can be calculated using an empirical formula. The maximum displacement at the center (w_c) of a rectangular diaphragm (with the dimension of *a* x *b*) under a uniform pressure of *P* is

$$w_c=\frac{1}{C_1}\frac{Pb^4}{D} \qquad (3)$$

with the value of the proportional constant C_1 determined by the ratio of *a* to *b*. Its value can be found by using the look-up table (see Table I). The stress at the center point of the long edge and the stress in the center of the plate are

$$\sigma_{\max}=\beta_1\left[\frac{b}{h}\right]^2 P \quad \text{and} \quad \sigma_c=\beta_2\left[\frac{b}{h}\right]^2 P \qquad (4)$$

where β_1 and β_2 are coefficient dependent on ratio *a* to *b* that can be found in the literature [6].

In order to get a valuable and precise solution, the analysis is applied with the maximal deflection by means of analyzing the following equation [3]:

$$C_1\frac{D}{b^4}w_c+C_2\frac{\sigma_0 h}{b^2}w_c=P+P_{w0} \qquad (5)$$

that includes the membrane residual stress and initial deflection. The right-side second term represents an equivalent force corresponding to the initial deflection from the internal mechanical strains of the membrane materials structure. The left-side second term represents the residual stress of the membrane. C1 and C2 are coefficients, depending on material anisotropy and geometric sizes. For two orientation of silicon they are equal [4][5]:

TABLE I.
LOOK-UP TABLE FOR COEFFICIENTS C_1 AND C_2

Coefficient		C_1			C_2
Material		Isotropic	Silicon <100>	Silicon <110>	-
b/a	1	792.449	866.36	728.75	15.35
	1.5	455.894	488.32	427.76	11.42
	2	394.552	410.58	380.48	10.34
	2.5	382.591	389.44	376.51	9.88
	3	380.325	381.98	378.35	9.55
Circular		64			4.45

III. MONTE CARLO ANALYSIS

The analysis was performed for membranes with dimension of 200 µm width and 4 µm of thickness. Membranes length varies respectively to the membrane ratio *b/a* presented in the Table I. As input parameters the dimensions and residual stress variation were chosen. The length and width varies up to 2 µm. The error is usually produced during the lithography process and mask fabrication. The thickness varies up to 0.2 µm due to inaccurate etching process. The residual stress varies from -10 MPa to 10 MPa that is a typical value for membrane fabricated in bonding process or for membranes covered with other layers (insulation, biocompatibility etc.). The analysis was performed for one million samples of input parameters and assumes the normal distribution. As an output parameter the membrane spring constant was chosen that is calculated using following formula:

$$k_{membr} = C_1 D / b^4 + C_2 \sigma h / b^2 \quad (6)$$

Below the modeling program's source code in a Matlab language is listed:

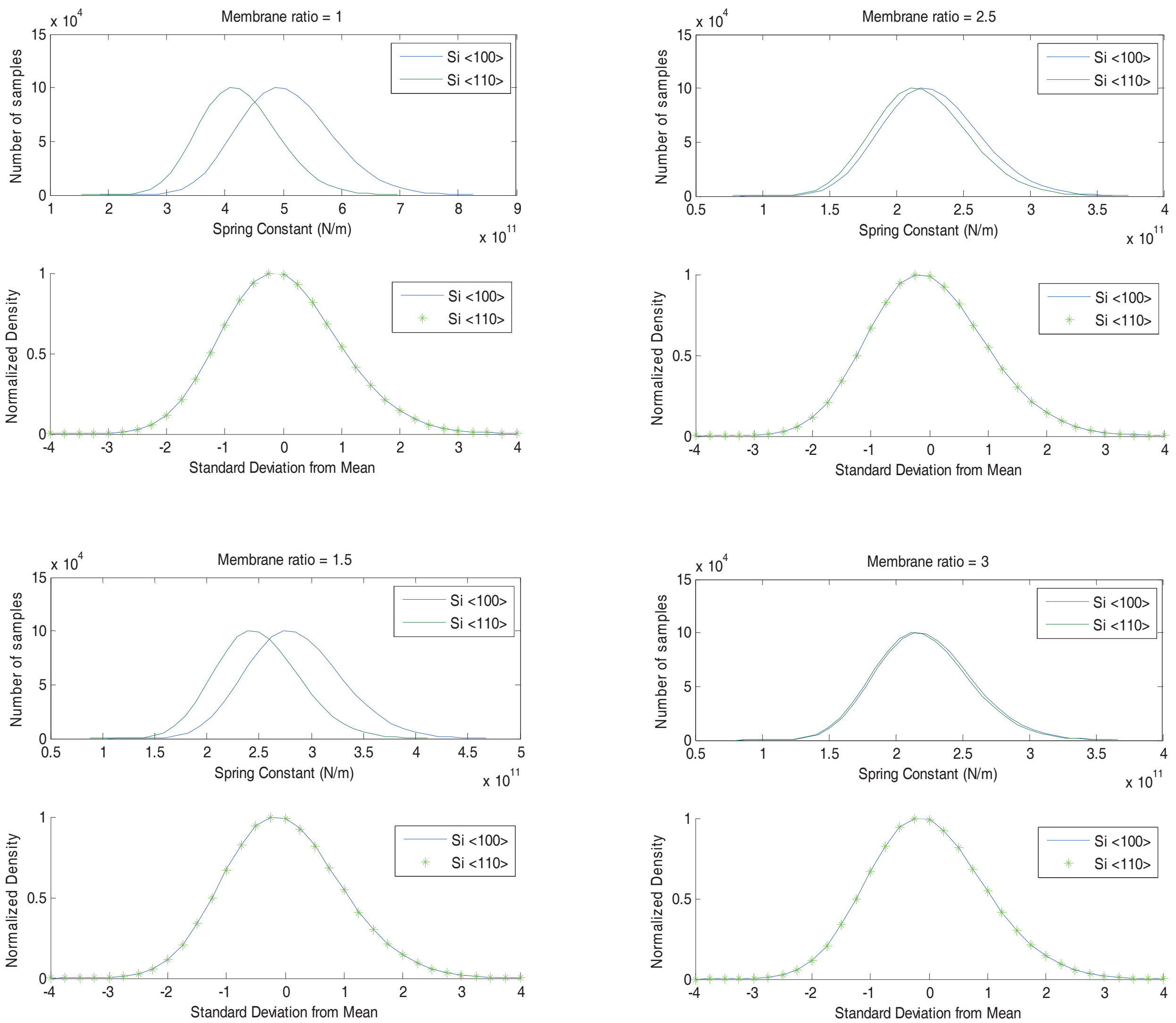
```
%%% clears the variables
clear all;
close all;
clc;
a = 200;                   % microns
b = 200;                   % microns
t = 4;                     % microns
str = 0e+6;              % MPa
%% The variances parameters of membrane
a_v = 2;                   % microns
b_v = 2;                   % microns
t_v = .2;                  % microns
str_v = 10e+6 ;            % MPa
%% Creates the random variables to use
r_a = randn(1,1e6);
r_b = randn(1,1e6);
r_t = randn(1,1e6);
r_str = randn(1,1e6);
%% Creates the vectors to use in calculating k
a1 = a + a_v*r_a;
t1 = t + t_v*r_t;
b1 = b + b_v*r_b;
str1 = str + str_v*r_str;
%% Does that conversion thing %%%%%%%%
convert = 1e6;
b2 = b1/1e6;
a2 = a1/1e6;
t2 = t1/1e6;
%%  Declares  the  elastic  modulus  as  a  material
property
E = 160e+9;     %MPa
v=0.279;
str=10e+6;
C1=  [866.36 488.32 410.58 389.44 381.98];
C1_2=[728.75 427.76 380.48 376.51 378.35];
C2=  [15.35  11.42  10.34  9.88   9.55];
ratio = [1 1.5 2 2.5 3];
for i=1:5
    k1=1*(E.*C1(i).*t2.^3)./(12*(1-v.^2)*(b2).^4)
+C2(i).*t2.*str1./(b2).^2;
    k2=1*(E.*C1_2(i).*t2.^3)./(12*(1-v.^2)*(b2).^4)
+C2(i).*t2.*str1./(b2).^2;
    variance = var(k1);  variance2 = var(k2);
    S1 = variance^.5;   S2 = variance2^.5;
    x1 = (mean(k1) - 4*S1):S1/4:(mean(k1) + 4*S1);
    x2 = (mean(k2) - 4*S2):S2/4:(mean(k2) + 4*S2);
    [h,x1] = hist(k1,x1);
    [hh,xx] = hist(k2,x2);
    figure;
    %% plots the histogram
    %subplot(2,1,1),plot(x1,h/max(h),xx,hh/max(hh));
    subplot(2,1,1),plot(x1,h,xx,hh);
    title(['Membrane ratio = ',num2str(ratio(i))]);
    xlabel('Spring Constant (N/m)')
    ylabel('Number of samples')
    legend('Si <100>','Si <110>');
    %%  creates  the  vector  of  standard  deviation  for
plotting
    y = -4:.25:4;
    h2 = h/max(h); hh2=hh/max(hh);
    subplot(2,1,2),
    hold on;
    plot( y, h2, 'b-', y, hh2, 'g*' );
    xlabel('Standard Deviation from Mean')
    ylabel('Normalized Density')
    legend('Si <100>','Si <110>');
end
```

IV. RESULTS

The results of Monte Carlo calculations are available on the Fig. 1. The calculations were performed for two crystal orientation respectively to the Table I. The upper parts of Fig. 1 present the distribution of spring constant value. The lower parts of Fig. 1 present the square deviation of spring constant from the mean value $M[k_{membr}]$ described with following formula :

$$\sigma_k = \sqrt{Var[k_{membr}]} \quad (7)$$

where *Var* is the variance.



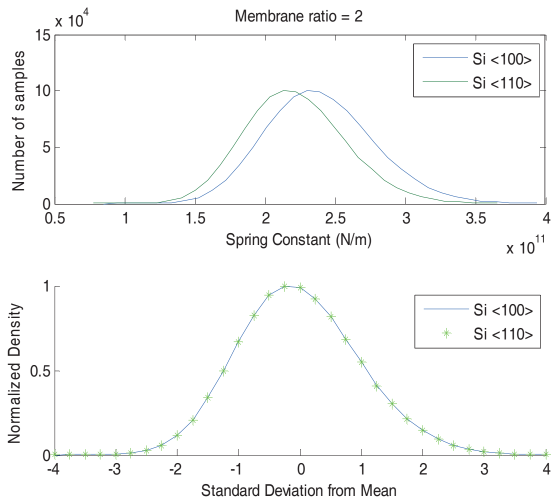


Fig.1. Calculated control number of spring and normalized density for membrane spring constant (upper figure) and standard deviation from mean value (lower figure) for various membrane *a*/*b* ratio

The results presented above show that the highest spring constant occurs for the square membrane. Also in this case the highest difference between spring constant values for two crystallographic orientations is observable: the average spring constant for <100> orientation is around 20% higher than for <110> orientation. When increasing the membrane length-to-width ratio, the value of spring constant decreases significantly. For the ratio of 1.5, it has a value almost 50% lower than in case of square membrane. Then it still decreases but near the ratio of 3 it becomes constant (what is known as a property of a long membrane). Moreover, it can be observed that with the increasing of the membrane ratio, the difference between two orientations seem to disappear. For the last case, with membrane ratio equal to 3, there is basically no difference between the results obtained for both orientations.

As far as the probabilistic distributions are concerned, they are basically the same for all cases. The distribution of membrane spring is similar to normal as it is linearly dependent on all input parameters. For all cases the standard deviation is equal to 16% of mean value. It may be concluded that most times the spring constant value is located within 2 standard deviations from mean (95%). Within 1 standard deviation is located about 70%. Therefore, a very small deviation in dimensions of the membrane leads to a relatively high change in spring constant. Let us now take into consideration the device that is based on the membrane. In a typical technological process the properties may be different for each membrane fabricated on the same wafer. Therefore, it is not possible to guarantee predicted performance for each device. Some devices will have worse performance and some should be rejected as they completely do not meet the requirements.

V. CONCLUSIONS

The statistical approach is very important in many domains of knowledge. In case of micromachined membranes it shows that even very simple devices may have significantly different performance than the expected one. The sources of errors seems to be negligible as the technological process allows fabrication of micromembranes with very high resolution. However, the simulations showed that the deviation from mean value is relatively high and many of devices will not meet the requirements. Therefore, the statistical simulation should be always performed in order to estimate the deviation of the device performance. Moreover, the results shows that the sensibility in many sensing devices based on the membrane will vary depending on the sample. Thus, each device should be properly calibrated. Thanks to the statistical simulations, it is possible to predict the yield production. Therefore, one can estimate the cost of the device before the fabrication what will be very useful to achieve the project success.

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