Statistical Optimization of the Cantilever Beams

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Abstract: **There are developed Matlab® code for optimize the cantilever geometric dimension for sensors of force field measuring.**

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I. INTRODUCTION

The oscillating system with concentrated elastic (elastic body stiffness μ) and inertial (working body mass m) parameters is a basic model for designing sensors of external floated physical fields of the cantilever beam type. The phase portrait of the dynamic state of the oscillatory systems of the cantilever beam – nanoobjects type with interaction potential $U(r)$ is simulated by the system of equations:

$$\begin{cases} U(r) = 4\varepsilon\left([\sigma/r]^{12} - [\sigma/r]^{6}\right), \\ W = d(W_k + eU(r))/dt, \\ \qquad p = mdr/dt, \end{cases} \tag{1}$$

where: ε – the so called "deep potential well" on the Lennard – Jones curve; r – distance vector; W_k – kinetic energy.

At distance $W \to 0$ it becomes non equal to zero while cantilever beam comes closer to a nanoobject. These sensors can record the external influences on the micro and nanoscale [1,2], to develop methods for measuring physical parameters of materials [3-7], to investigate the electric field distribution near the surface of nanoobjects [8,9].

In this system, the field interaction between cantilever beam and analysis nanoobjects is simulated by linear spring with stiffness μ, corresponding to the linearization of Lennard – Jones potential in static equilibrium state. When the location from cantilever beam to test object is far, then it takes a horizontal position, when approaching an object, the cantilever beam begins to deform, but at some distance from the object it takes a horizontal position – this is a static equilibrium [10].

The amplitude of the system is determined damping properties, which are determined by Young's modulus and the geometrical dimensions of the suspension. Young's modulus for a particular task is usually defined by the micro device designer, so for ensuring the optimal oscillation amplitude of the inertial mass is optimize the geometric dimensions.

The measurements are based on the frequency of resonance oscillations of the cantilever beam or cantilever beam. For systems with concentrated linear elasticity (rectangular console beam with width h, thickness t and length L) and inertia, square of resonance frequency, the objective function of optimization for oscillatory system:

$$\omega_0^2 = \frac{\mu}{m} = \frac{E_s t^2}{4L^4\rho}, \tag{2}$$

where μ is the spring constant of the rectangular cantilever.

It will be assumed that there is no change in the spring constant. m is the original effective mass, t is the beam thickness, L is length of the cantilever, ρ is beam density, E_s is Young's modulus, I is area moment of inertia. Moment of inertia can be expressed as:

$$I = \int y^2 dS, \tag{3}$$

where dS is the infinitesimal area element; y is the distance of this area element to the origin interest.

It seems that random changes in the physical and geometrical parameters of the oscillator will lead to fluctuations of resonant frequency, and thus to increase measurement error. That is why one of the urgent problems is the optimization of geometric parameters of the cantilever beam.

II. PHYSICAL AND MATHEMATICAL MODEL

In Figure 1, the scheme of cantilever beam is presented.

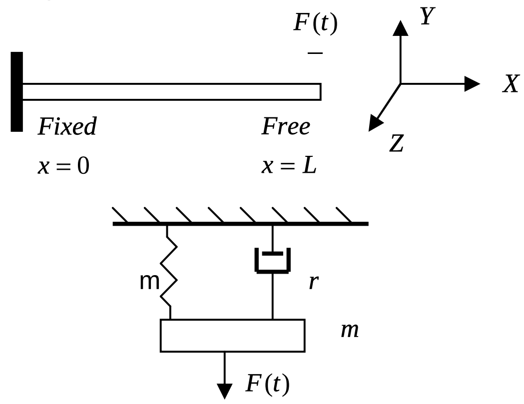

Fig.1. The rectangular cantilever beam.

The Euler – Bernoulli equation provides a first order description of the deflection $w(x,t)$ of an elastic beam [1]:

$$\frac{\partial^4 w}{\partial x^4} + \frac{\rho S}{E_s I}\frac{\partial^2 w}{\partial t^2} = 0 , \qquad (4)$$

where: S is beam cross – sectional area.

Bound conditions for a cantilever beam equal:

1) fixed at: $x = 0$, $w(0,t) = 0$, $\frac{\partial w(0,t)}{\partial x} = 0$. (5)

2) free at: $x = L$, $\frac{\partial^2 w(L,t)}{\partial x^2} = 0$, (6)

$$-E_s I \frac{\partial^3 w(L,t)}{\partial x^3} = F(t) - m\frac{\partial^2 w(L,t)}{\partial t^2} . \qquad (7)$$

Initial conditions equal:

$$w(x,0) = 0 , \quad \frac{\partial w(x,0)}{\partial t} = 0 . \qquad (8)$$

Deflection free end beam equal:

$$deflection = -\frac{F}{6E_s I}(3Lx^2 - x^3) . \qquad (9)$$

III. STATISTICAL OPTIMIZATION

Let us connect the optimization of the geometrical parameters, considered above, with the constraints of nature. According to [11,12], constraints conditions should be included into program structure:

$$\frac{FL}{I} y = \sigma < \sigma_{max} = \frac{3E_s t}{2I} , \qquad (10)$$

which take into account, that the most mechanical stresses during the cantilever beam bending appears onto the opposite surfaces (top surface stretches, bottom shrinks), which are placed at a distance

$$y_{max} = \frac{t}{2} , \qquad (11)$$

from the neutral axis without mechanical stresses along it. Here σ is stress in the beam, σ_{max} is the maximum bending stress .

The Matlab® code for optimization of silicon cantilever beam values h_{opt} and L_{opt} is presented:

```
clear all;
close all;
clc;
FOS=5; % Factor of Safety
F= 1; % Force
t=5; % thickness cantilever mkm
E_sigma=190e3; % Youngs Modulus,[13]
sigma_Yield=70; %[13]
def_max= 1; % Maximum allowable deflection
stept=0.01;
tmin=2;
tmax=15;
Lmin= 20; % Minimum length cantilever
Lmax=500;% Maximum length  cantilever
stepL=0.1;
Eq1= 6*F*FOS/(t^2*sigma_Yield);
Eq2=4*F/(E_sigma*def_max*t^3);
L= Lmin:stepL:Lmax;
[m,n]=size(L);
h1=5;
for j=1:1:n
h1(j)=Eq1*L(j);
end
h2=5;
for j=1:1:n
h2(j)=Eq2*(L(j)^3);
end
plot(L,h1,L,h2);
grid on;
xlabel('L (lenght canteliver),mkm');
ylabel('h1,h2(width canteliver),mkm');
title('Plot for two constraint conditions')
ylim([0 20])
 figure(2);
[t1,L1]=meshgrid(tmin:stept:tmax,Lmin:stepL:Lmax);
 hsurf1=L1*6*F*FOS./(t1.^2*sigma_Yield);
hsurf2=(L1.^3)*4*F./(E_sigma*def_max*t.^3);
 surf(L1,t1,hsurf1);
shading interp;
hold on
surf(L1,t1,hsurf2);
view([60 20]);
```

Analysis of the literature indicates a considerable data scatter relatively to the value of parameter sigma_Yield. Values of the relevant parameters have taken from work [13]. Since silicon is a brittle material, the ultimate strength value is taken for calculating the factor of safety (FOS = ultimate strength/maximum stress) design margin over the theoretical design capacity. It is shown in Figure 2, that optimized values h_{opt} and L_{opt} defined as the coordinates of the curves intersection point equal $L_{opt} = 320\,\mu m$ and $h_{opt} = 5.5\,\mu m$.

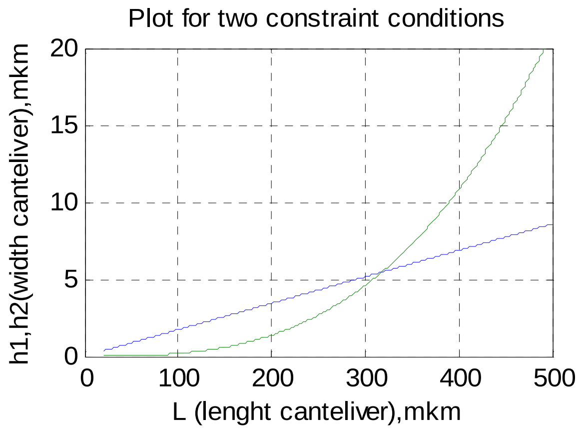

Fig.2. Plot of the two curves *h1*(*L*) and *h2*(*L*).

First of the coordinate described by the equation

$$h1 = \frac{6 \cdot L \cdot F \cdot FOS}{t^2 \cdot \sigma} , \qquad (12)$$

and second of the coordinate described by the equation

$$h2 = \frac{4 \cdot L^3 \cdot F}{t^3 \cdot E_s \cdot \delta_{max}} , \qquad (13)$$

where maximum deflection

$$\delta_{max} = \frac{F}{3E_s I} L^2 . \qquad (14)$$

The wider cantilevers allowing increasing the value of the marginal mechanical stress. Parameters of the standard rectangular cantilever: length 200 μm, thickness 1-5 μm, width 30 μm, weight about 35 ng and stiffness order 37 N/m, so the resonant oscillation frequency about 10-400 kHz [14], while by the following formula:

$$\omega_{rez} = \sqrt{\frac{\mu}{m}} = \sqrt{\frac{37}{35\times10^{-9}}} \cong 32.5 kHz. \qquad (15)$$

In Figure 3 is shown the 3D-figure with intersection of the two surfaces *h1*(*L*,*t*) and *h2*(*L*,*t*). The optimized values of parameters h_{opt}, L_{opt}, t_{opt} is situated in points of intersection.

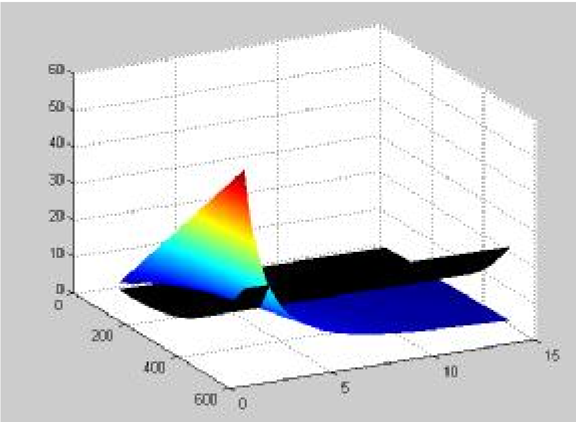

Fig.3. Plot of the two surfaces *h1*(*L*,*t*) and *h2*(*L*,*t*).

IV. Conclusion

In this paper was created the program of optimization geometric dimensions of the cantilever beam in order to build in further work the maps of fluctuation spectrum of the oscillation amplitude of console with random changing of external influence on it.

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