MATLAB-Simulink Model of Mechanical Component of Fully Differential Capacitive MEMS Accelerometer

Andriy Holovatyy[1], Vasyl Teslyuk[2], Mykhaylo Lobur[3]

1. Software Engineering Department,Ternopil Ivan Pul'uj National Technical University, UKRAINE, Ternopil, Ruska street 56, E-mail: aholovatyy@yahoo.com
2. CAD Department, Lviv Polytechnic National University, UKRAINE, Lviv, S. Bandery street 12, E-mail: vtesliuk@polynet.lviv.ua
3. CAD Department, Lviv Polytechnic National University, UKRAINE, Lviv, S. Bandery street 12, E-mail: mlobur@polynet.lviv.ua

Abstract - **In the paper, simulation model of the mechanical component of the fully differential capacitive MEMS accelerometer has been developed using MATLAB/Simulink. The model allows to simulate movement of the proof mass, capacitance changes of the measuring capacitors of the sensitive element, sensitivity of the sensor depending on the applied force of acceleration, and to perform the transient analysis of the integrated device at the system level of computer-aided design.**

Keywords - **Microelectromechanical systems (MEMS), fully differential capacitive MEMS accelerometer, acceleration, mathematical modeling, simulation model, transient analysis, CAD system, MATLAB/Simulink.**

I. Introduction

Microelectromechanical systems (MEMS) are miniature integrated devices produced by using micromachining technologies. They combine mechanical and electrical components. Mechanical components can be classified into sensors, devices that convert any physical motion into electrical signal, and actuators that perform the opposite action. They convert electrical energy to mechanical motion. Nowdays, MEMS technologies are developed very rapidly, because of the demand of such devices in different engineering areas. One of such devices is a sensor for measuring acceleration (integrated accelerometers (MEMS accelerometers)). MEMS accelerometers play a sufficient role in modern techniques and they are widely used in automobiles (airbag deployment systems. roll over detection, electronic stability control, navigation, security systems and active suspension), consumer electronics (smartphones, laptops, tablets), sporting equipment, military applications and etc [1].

The important role in the design of such heterogeneous systems as MEMS accelerometers plays computer-aided design systems that allow to decrease development time of the integrated devices and to decrease their cost. The conducted analysis of the papers on MEMS design allows to say that creation of qualitatively new mathematical and computer models of the integrated accelerometers for optimization of their construction parameters with technical characteristics and improvement of design efficiency is an actual task. [2-4]

II. Construction and Mathematical Model of Mechanical Component of Fully Differential Capacitive MEMS Accelerometer

In Fig. 1 schematic view of the construction and mechanical parameters of the SE of the fully differential capacitive MEMS accelerometer are shown. The central part of the SE of MEMS accelerometer is a proof mass M, which is suspended by the spring elements with a spring coefficient K to the frame

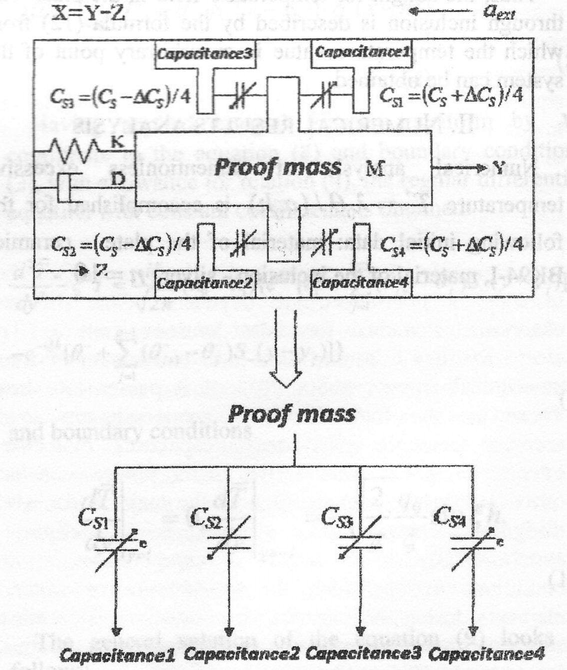

Fig 1.Schematic view of the SE of the fully differential capacitive MEMS accelerometer

Movement of the proof mass can be described by the 2nd order differential equation (DE), as a system mass-spring-damper:

$$M\frac{d^2x}{dt^2}+D\frac{dx}{dt}+K_{eff}x(t)=F_{ext}(t)=Ma_{ext}(t) \quad (1)$$

where M – mass of the proof mass, D and K_{eff} - damping coefficient and spring constant respectively, F_{ext} – external force of inertia which acts on the proof mass due to the external acceleration a_{ext}.

Analytical solution of such nonhomogeneous DE (1) will be a sum of the solutions of the general solution of its homogeneous equation $x_c(t)$ and partial solution of the DE equation (1) $X_p(t)$: $x(t)=x_c(t)+X_p(t)$.

The general solution will be the following:

$$x_c(t)=c_1e^{\alpha t}\cos(\beta t)+c_2e^{\alpha t}\sin(\beta t) = e^{\alpha t}(c_1\cos(\beta t)+c_2\sin(\beta t)), \text{ or}$$

$$x(t)=ae^{\alpha t}\cos(\beta t-\varphi) \quad (2)$$

where $a=\sqrt{c_1^2+c_2^2}$ i φ – amplitude and phase shift of the movement of the SE.

If $\omega=\beta$, then the partial solution of the DE (1) has the following form:

$$X_p(t)=At\cos(\omega_0 t)+Bt\sin(\omega_0 t)$$

$$(-M\omega_0^2+K_{eff})At\cos(\omega t)+(-M\omega_0^2+K_{eff})Bt\sin(\omega t)$$

$$+2M\omega_0B\cos(\omega t)-2M\omega_0A\sin(\omega t)=F_0\cos(\omega t)$$

$$X_p(t)=\frac{F_0}{2M\omega_0}t\sin(\omega_0 t) \quad (3)$$

Thus, the final solution of the DE (1), when $\omega=\beta$, has the following form:

$$x(t)=ae^{\alpha t}\cos(\beta t-\varphi)+\frac{F_0}{2M\omega_0}t\sin(\omega_0 t) \quad (4)$$

The designing MEMS accelerometer has a fully differential topology. It means that there are four measuring capacitances C_{s1}, C_{s2}, C_{s3}, C_{s4} :.

$$C_{s1,2}=(C_s\pm\Delta C_s)/4 \quad (5)$$

where C_s – nominal capacitance at rest and ΔC_s the change of the nominal capacitance of the microsensor. Under the action of the inertia force the proof mass moves along the sensitivity axis respectively to the moving coordinate system (X = Y – Z), that cause to the distance change between its comb-finger electrodes and nonmoving comb-finger electrodes of the integrated device.

If a_{ext} = 0, then capacitances of the measuring capacitors are equal and can be determined by the formula:

$$C_{s1}=C_{s2}=C_{s3}=C_{s4}=A_c\varepsilon_r\varepsilon_0/d=C_s/4 \quad (6)$$

where A_c – area of the measuring comb-finger electrode; d – distance between comb-finger electrodes (plates) of the capacitor; ε_r – dielectric permittivity of the environment between capacitor plates; ε_0 – vacuum permittivity ($8{,}8541\times10^{-12}$ F/m).

If $a_{ext}\neq 0$ and $\Delta d << d$, then the change of the capacitances of the measuring capacitance can be calculated by the following formula:

$$\Delta C_s=C_{s1}+C_{s4}-C_{s2}-C_{s3} = 2A_c\varepsilon_r\varepsilon_0\left(\frac{1}{d-\Delta d}-\frac{1}{d+\Delta d}\right)\approx C_s\frac{\Delta d}{d} \quad (7)$$

and the capacitances of the capacitors are calculated by the formulas:

$$C_{s1}=C_{s4}=(C_s+\Delta C_s)/4$$

$$C_{s2}=C_{s3}=(C_s-\Delta C_s)/4 \quad (8)$$

From (7) and $\frac{x}{a_{ext}}\approx\frac{1}{\omega_0^2}$ the capacitive sensitivity of the accelerometer can be calculated by the formula:

$$S_c=\frac{\Delta C_s}{a_{ext}}=\frac{C_s}{d}\frac{M}{K_{eff}}=\frac{C_s}{d}\frac{1}{\omega_0^2} \quad (9)$$

The sensitivity of the system can be calculated by the formula:

$$S_e=\frac{V_{out}}{a_{ext}}=\frac{K_v}{\omega_0^2} \quad (10)$$

where V_{out} – output voltage which can be calculated from the formula $V_{out}=V_m\cdot\left(\frac{C_{s1}-C_{s2}}{C_{s1}+C_{s2}}-\frac{C_{s3}-C_{s4}}{C_{s3}+C_{s4}}\right)$, V_m – modulation voltage.

III. MATLAB-Simulink Model of the Mechanical Component of the Fully Differential Capacitive MEMS Accelerometer

In Fig. 2 the developed MATLAB-Simulink simulation model of the mechanical component of the fully differential capacitive MEMS accelerometer is shown.

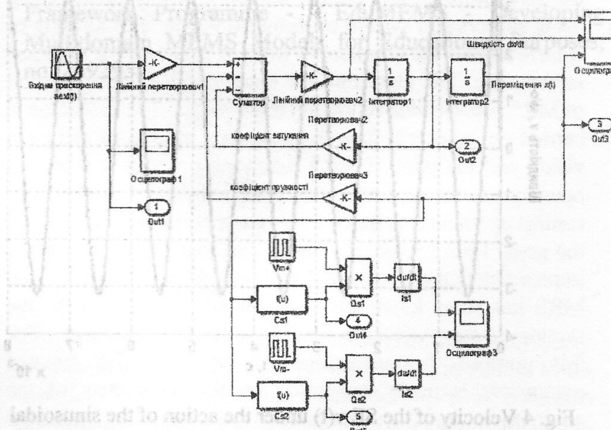

Fig. 2 Simulink – model of the mechanical component of the fully differential capacitive MEMS accelerometer

In Fig. 3,4 simulation results of the movement and velocity of the proof mass of the SE at the sinusoidal acceleration in 5g. From the diagram it is seen that the displacement of the proof mass is in the range from -50 nm to 50 nm. In Fig.5 the diagram of the displacement of the proof mass under the action of the constant acceleration 5g. The diagram in Fig.6 shows the change of the capacitances of the measuring capacitors depending on the applied acceleration of 5g which is located within 627,8 … 629,2 fF. In Fig. 7 the dependence of the capacitive sensitivity on the working resonance frequency in the range from 5 to 100 Hz for the various values of the nominal capacitance is shown.

Therefore, for such construction parameters of the SE of such fully differential capacitive MEMS accelerometer the highly accurate amplifiers and high-sensitive electronic circuits for processing of such signals are required.

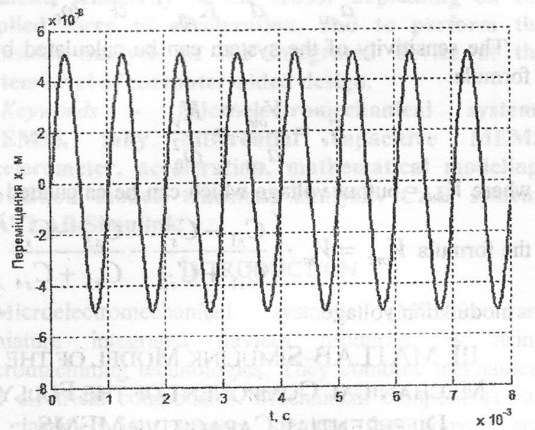

Fig. 3 Movement of the SE x(t) under the action of the sinusoidal acceleration 5g

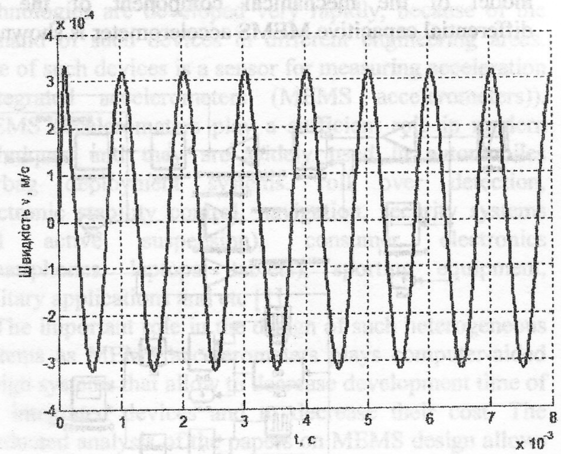

Fig. 4 Velocity of the SE v(t) under the action of the sinusoidal acceleration 5g

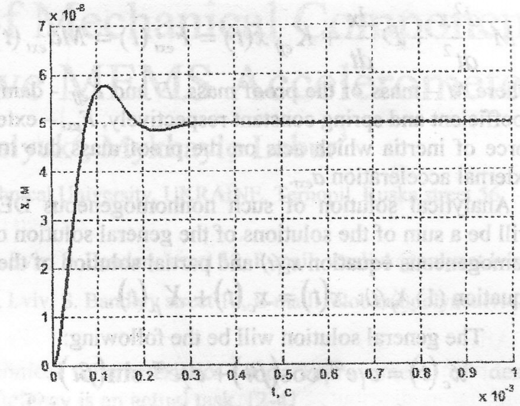

Fig. 5 Displacement of the SE under the action of the acceleration 5g

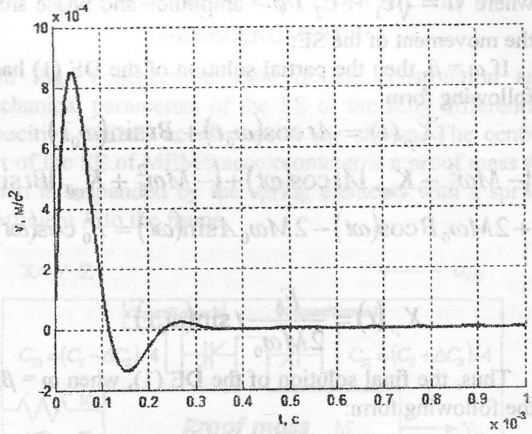

Fig. 6 Velocity of the SE response under the action of the acceleration *5g*

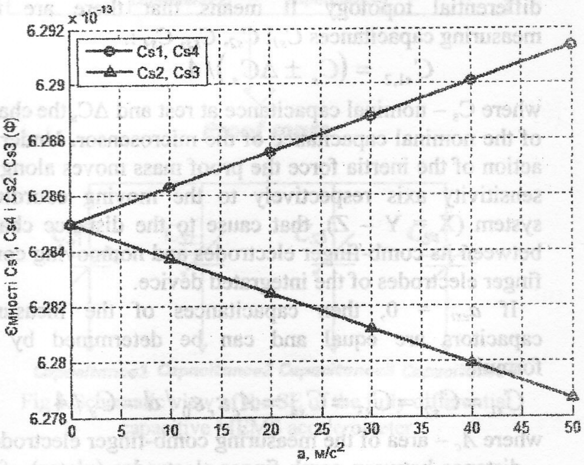

Fig. 7 Capacitance changes C_{s1}, C_{s4} and C_{s2}, C_{s3} depending on the applied acceleration a(t)

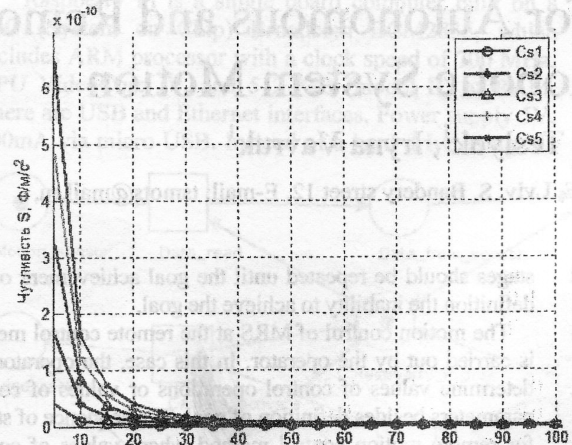

Fig. 8 Sensitivity dependence on the resonance frequency S_e, F/m/s^2 for the various values of the nominal capacitance

IV. Conclusion

The MATLAB/Simulink simulation model of the mechanical component of the fully differential capacitive MEMS accelerometer has been developed. On the base of the developed model the simulation of the behavior of the SE of the fully differential capacitive MEMS accelerometer has been performed. From the obtained simulation results the plots of dependencies are depicted and analysis of the output dynamic characteristics of the SE of MEMS accelerometer on the parameters of its construction, and also movement of the working mass, change of the output measuring capacitances of the SE and sensitivity on the resonance frequency of the SE for the various values of the nominal capacitance of the microsensor for its defined construction parameters under the action of the external acceleration.

References

[1] N. Yazdi, F. Ayazi, and K. Najafi. “Micromachined Inertial Sensors”, *Proc* IEEE,Aug. 1998,Vol. 86, No. 8.

[2] Kanchan Sharma, Isaac G. Macwan, Linfeng Zhang, Lawrence Hmurcik, Xingguo Xiong. Design Optimization of MEMS Comb Accelerometer, Department of Electrical and Computer Engineering, pp. 10.

[3] Tolga Kaya, Behrouz Shiari, Kevin Petsch and David Yates. Design of a MEMS Capacitive Comb-drive Accelerometer. Central Michigan University, 2012, pp. 6.

[4] Akila Kannan. Design and Modeling of a MEMS-Based Accelerometer with Pull In Analysis. Thesis, University of British Columbia, 149, pp. 149.

[5] Dr. Lynn Fuller, Motorola Professor. MEMS Capacitor Sensors and Actuators. / Dr. Fuller Lynn. – Rochester Institute of Technology, Microelectronic Eng.. – May, 2007. – 34 p.

[6] W. Yun. A Surface Micromachined Accelerometer with Integrated CMOS Detection Circuitry : Doctoral Dissertation. Univ. of California at Berkeley, 1992.

[7] Fedder G. K. Simulation of Microelectromechanical Systems Doctoral Dissertation / G.K. Fedder. – Univ. of California at Berkeley, 1994.

[8] Pelesko John A. Modeling MEMS and NEMS / John A. Pelesko and David H. Bernstein. Chapman & Hall/CRC, 2002, 384 p.

[9] Frank W. Pietryga. Solving Differential Equations Using MATLAB/Simulink. University of Pittsburgh at Johnstown, 2005, 11 p.

[10] David Houcque. Introduction to Matlab for engineering students. Northwestern University. 2005, 74 p.

[11] Matlab Programming Fundamentals R2013b. The MathWorks, Inc. 2013, 985 p.

[12] Matlab – The Language of Technical Computing. The MathWorks, Inc. 2002, 602 p.

Acknowledgements

Results presented in the paper are supported by Marie Curie International Research Staff Exchange Scheme Fellowship within the 7th European Community Framework Programme - - EduMEMS - Developing Multidomain MEMS Models for Educational Purposes, no. 269295.