Modeling and Behavioral Simulation of Electro-Thermal Microactuator using VHDL-AMS

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Abstract – **In the paper, VHDL-AMS model of electro-thermal microactuator for computer-aided design is created using** hAMSter**. The created model allows to simulate the dependence of the flexure of hot (thin) and cold (wide) arms on their lengths and the applied voltage between the anchors of the microactuator, dependence of the current on the applied voltage and the arm length of the microactuator, temperature distribution along the arms, dependence of the power consumption on the applied voltage, and also to perform the behavioral analysis of this device at the functional design level.**

Keywords - **MEMS, electro-thermal microactuator, Joule-Lenz's law, VHDL-AMS model, computer-aided design, hAMster HDL simulation software.**

I. INTRODUCTION

MicroElectroMechanical Systems (MEMS) – devices having dimensions from 20 µm to 1 mm, that combine microelectronic and micromechanical components, and which are fabricated using Integrated Circuit (IC) technology. One of the critical MEMS components is a microactuator – device which converts energy into mechanical motion. The application of the microactuators is very broad and various and also constantly increases. They are used in robotics, control devices, aerospace engineering, biomedicine, dosimetry, in measurement instrumentation, game technologies, automotive industry etc. The main mechanisms used for receiving the actuation (motion, deflection and force) in such devices are the following: electrostatic, magnetic, piezoelectric, hydraulic and thermal. Since electrothermal actuation gives the simple controllable actuation mechanism which is compatible with standard microelectronics, therefore electrothermal microactuators are one of the most used and perspective control devices in MEMS [1-3].

One of the important stage of the MEMS device design is its mathematical modelling. On the marketplace for software, there are a lot of software tools for computational modeling of integrated devices and manufacturing methods (engineering processes). Existing universal software tools most frequently are environments with a wide range of functions and methods for solving algebraic and differential equations, and with graphic illustration of obtained results. The use of such universal software tools efficiently enough allows to solve scientific-research problems, but it is not always suitable for obtaining solutions with a required accuracy during a short time interval [6].

Since the design of the MEMS devices includes information from such physical areas of knowledge as electronics, mechanics, electrostatics, thermodynamics, therefore their full static and dynamic analysis is very complex. Practical solution is creation of simplified models in hardware description languages such as VHDL-AMS, Verilog-AMS, MAST with the further use of special software. For example, such as: Synopsys Saber, MATLAB, Cadence, hAMSter, SMASH (Dolphin Integration) and others [7-9].

Thus, development of behavioral models of the MEMS devices for their simulation and analysis is an actual task.

II. CONSTRUCTION AND PRINCIPLE OF OPERATION OF ELECTRO-THERMAL MICROACTUATOR

The construction of the traditional two-arm electro-thermal microactuator consists of a thin ("hot") arm, wide ("cold") arm and flexure arm connected together at one end constrained elastically at the anchors, which in turn are rigidly attached to the substrate (Fig. 1). The hot arm usually is thinner than the cold arm, therefore the electrical resistance of the hot arm is greater than the electrical resistance of the cold arm.

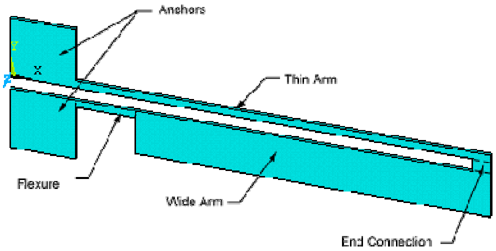


Fig. 1. Construction of MEMS electro-thermal actuator

The principle of operation of the two-arm electro-thermal microactuator uses the principle of Joule heating and is the following. Across the microactuator anchors the voltage is applied which creates electrical field. When electrical current flows through the cold and hot arms, heat generated in the hot arm is larger than the heat generated in the cold arm. Since the cold and hot arms are made of the same material and have the same thermal expansion coefficient, so the temperature difference causes the larger expansion of the hot arm than the cold arm. As the result of such expansion is the deflection (motion) of the microactuator tip towards the cold arm.

III. MATHEMATICAL MODEL OF ELECTRO-THERMAL MICROACTUATOR

The heat dissipated through convection and radiation to the ambient can be neglected for the analysis. Since the length of each arm is much larger than its width and height, a two-dimensional model is developed to simplify the analysis. According to the principle of heat transfer, heat conduction can be determined by:

$$Q = -K_p S \frac{\partial T}{\partial x} \tag{1}$$

where K_p is the thermal conductivity of polysilicon, S is the conductive cross-sectional area, T is the temperature and x is the length of the structure. If heat is transferred into the structure and then out of the structure, the heat conduction into the structure is:

$$Q_t = -K_p wh \left(\frac{\partial T}{\partial x}\right)_x \tag{2}$$

while the heat conduction out of the structure is:

$$Q_0 = -K_p wh \left(\frac{\partial T}{\partial x}\right)_{x+dx} \tag{3}$$

where w is the width and h is the height of the microactuator. When voltage is applied between the two terminals of this thermal microactuator, current is passed through the entire system and travels from one anchor to the other. This results in the joule heat (4).

$$Q_J = j^2 \rho wh dx \tag{4}$$

$$\rho = \rho_0 \left[1 + \xi \left(T - T_0\right)\right] \tag{5}$$

$$j = \frac{V}{\rho L} \tag{6}$$

where j is the current density, ρ is the resistivity of the structure, ρ_0 is the resistivity of the beam at temperature T_s, T_s is the substrate temperature, ξ is the temperature coefficient of resistance, V is the voltage across the arm and L is the length of the arm. According to the first law of thermodynamics (conservation energy) :

$$Q_I + Q_J = Q_0 \tag{7}$$

Substituting Equations (2) to (4) into Equation (7), taking the limit as $dx \rightarrow 0$, produces the following second order differential equation:

$$K_p wh \left(\frac{\partial^2 T}{\partial x^2}\right) + j^2 \rho wh = 0 \tag{8}$$

Since the microactuator has the same conductive cross-sectional area, Equation (8) is reduced to Equation (9).

$$\left(\frac{\partial^2 T}{\partial x^2}\right) = -\frac{j^2 \rho}{K_p} \tag{9}$$

The boundary conditions assumed are that the anchor pads have the same temperature as the substrate, T_s, that is:

$$T(0) = T(L) = T_s \tag{10}$$

Solving Equation (9) and (10) gives the temperature distribution along the arm is:

$$T\left(x\right) = \frac{V^2}{2L^2 \rho K_p}\left(Lx - x^2\right) + T_s \tag{11}$$

From Equation (11), it is shown that the temperature is parabolic and symmetric about the center point of the length with a maximum temperature, T_m at $x = L/2$. By substituting $x = L/2$, the maximum temperature is:

$$T_m = \frac{V^2}{8\rho K_p} + T_s \tag{12}$$

When the thermal gradient perpendicular to the beam axis is imposed, the conduction heat transfer in a plane beam is:

$$T\left(x\right) = (T - T_s)x / L + T_s \tag{13}$$

$$T\left(y\right) = \frac{T_h - T_c}{h} y + \frac{T_h + T_c}{2} \tag{14}$$

Where T_s is the temperature of the bottom of the beam (cold arm temperature) and T_h is the temperature of the top of the beam (hot arm temperature). Equation (13) shows that the top of the beam mimics the hot arm of the microactuator while the bottom of the beam mimics the cold arm of the microactuator. By denoting the deflection in the *x-y* plane by $v(x)$, the steady-state deflection of an elastic beam is (1,6):

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2}\right) + P \frac{\partial^2 y}{\partial x^2} = F \tag{15}$$

Where E – Young's modulus of the beam, I – moment of inertia, v – deflection of the beam, P – load of the beam and F is the applied force. Assume there is no motion of the beam in the *y-z* plane. The applied force can be written as:

$$F = p\left(x\right) - \frac{\partial^2 M_{Tz}}{\partial x^2} \tag{16}$$

Assume that the load *p(x)* the contribution from the effects other than thermal stress is identically zero. The term M_{Tz} is computed by:

$$M_{Tz} = \int_S \alpha ETy dA \tag{17}$$

where M_{Tz} is the bending moment and α is the thermal expansion coefficient. From the Equation (17), the integral is an area integral over the cross-section of the beam. Using the assumed form for temperature and assuming that α and E are constant:

$$M_{Tz} = \frac{\alpha E w h^2}{12}\left(T_h - T_c\right) \tag{18}$$

The assumption that there is no motion in the *y-z* plane is required to assume the analogous quantity M_{Tz} defined as:

$$M_{Ty} = \int_S \alpha E T z dz \tag{19}$$

This equation is defined to be identically zero. This follows from the assumed form for the temperature field. The expression for M_{Tz} implies that:

$$\frac{\partial^2 M_{Tz}}{\partial x^2} = 0 \tag{20}$$

Substituting Equation (19) into Equation (15) gives F=0. Therefore, both Equations are reduced to:

$$EI\frac{\partial^4 y}{\partial x^4} + P\frac{\partial^2 y}{\partial x^2} = 0 \tag{21}$$

Further assume that P=0 and hence simplify to:

$$\frac{\partial^4 y}{\partial x^4} = 0 \tag{22}$$

Assuming the left end of the beam is held fixed or anchored, the boundary conditions can be applied:

$$v\left(0\right) = \frac{\partial y}{\partial x}\left(0\right) = 0 \tag{23}$$

Assume the right end of the beam is free or unsupported. In the presence of thermal stresses, the boundary conditions at the free end of a beam are [6, 7]:

$$EI\frac{\partial^2 v}{\partial x^2}(L) = -M_{Tz} \tag{24}$$

$$EI\frac{\partial^3 v}{\partial x^3}(L) + P\frac{\partial y}{\partial x} = -\frac{\partial M_{Tz}}{\partial x} \tag{25}$$

Using Equation (18), this can be reduced to:

$$EI\frac{\partial^2 v}{\partial x^2}\left(x\right) = -\frac{\alpha E w h^2}{12}\left(T_h - T_c\right) \tag{26}$$

The differentiation of this Equation (26) gives:

$$\frac{\partial^3 v}{\partial x^3}(L) = 0 \tag{27}$$

Taking the integration of the equation below and applying the boundary condition to find the beam deflection:

$$EI\frac{\partial^2 v}{\partial x^2}\left(x\right) = -\frac{\alpha E w h^2}{12}\left(T_h - T_c\right) \tag{28}$$

$$\frac{\partial^2 v}{\partial x^2}\left(x\right) = -\frac{\alpha w h^2}{12I}\left(T_h - T_c\right) \tag{29}$$

Finally, the deflection in Equation (28) is obtained as:

$$v\left(x\right) = -\frac{\alpha w h^2}{24I}\left(T_h - T_c\right)x^2 \tag{30}$$

where I is defined as:

$$I = \int_S y^2 dS = \frac{wh^3}{12} \tag{31}$$

Therefore, Equation (28) is reduced to:

$$v(x) = -\frac{\alpha}{2h}\left(T_h - T_c\right)x^2 \tag{32}$$

$$\begin{gathered} h = \left(\frac{L_{ch}}{L_h}\right)\left(w_h + w_c + L_g\right) + \left(\frac{L_c - L_{ch}}{L_h}\right)\left(w_h + w_c\right) + \\ + \left(\frac{L_f}{L_h}\right)\left(w_f + w_c\right) \end{gathered} \tag{33}$$

The power consumption can be estimated after finding the current flow through the thermal bimorph microactuator by using Equation (35).

$$R = \rho\left(\frac{L_h}{w_h h_h} + \frac{L_c}{w_c h_c} + \frac{L_f}{w_f h_f}\right) \tag{34}$$

$$P = IV \tag{35}$$

IV. DEVELOPMENT OF VHDL-AMS MODEL OF ELECTRO-THERMAL MICROACTUATOR

Design of MEMS devices on the functional-logic level design includes the creation of the mathematical model. Such mathematical models include information from the different scientific and engineering areas. For creation of such models, a special language - VHDL-AMS is used [5-8].

Extension of VHDL standard to VHDL-AMS (Very High Speed Integrated Circuits Hardware Description Language Analog-Mixed Signals) for description of digital-, analog- and mixed-signal models of devices, which use not only electrical signals, and also optical, chemical, thermal, mechanical and others, is an important stage for creation of the universal IC/MEMS CAD systems, that allows to make automatic compilation of topological solutions based on the high-level descriptions [6-8].

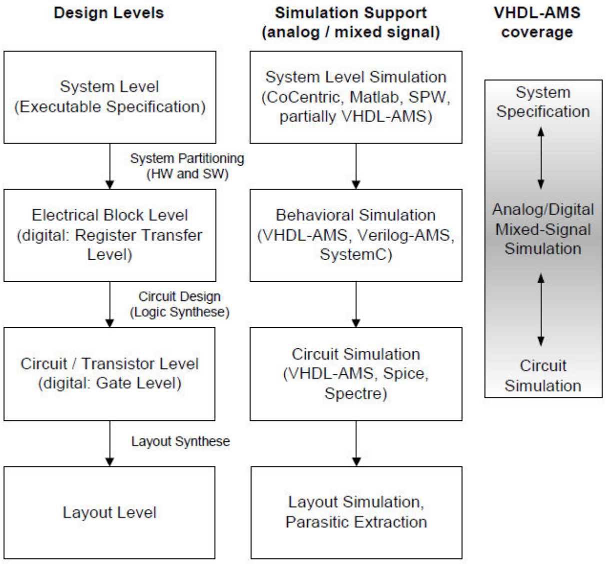
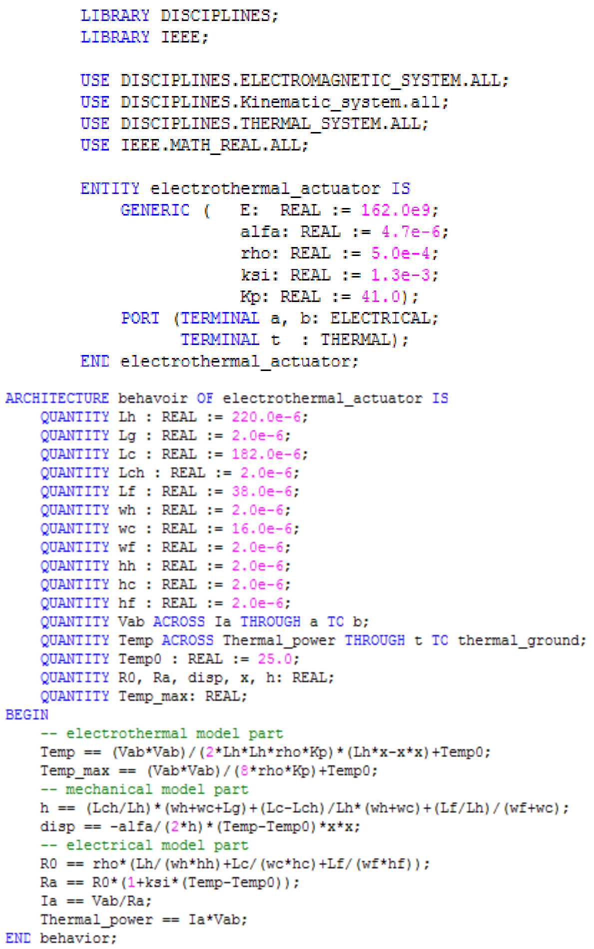


Fig. 2. Use of VHDL-AMS in the design of MEMS devices



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LIBRARY DISCIPLINES;
LIBRARY IEEE;

USE DISCIPLINES.ELECTROMAGNETIC_SYSTEM.ALL;
USE DISCIPLINES.Kinematic_system.all;
USE DISCIPLINES.THERMAL_SYSTEM.ALL;
USE IEEE.MATH_REAL.ALL;

ENTITY electrothermal_actuator IS
    GENERIC (   E:  REAL := 162.0e9;
                alfa: REAL := 4.7e-6;
                rho: REAL := 5.0e-4;
                ksi: REAL := 1.3e-3;
                Kp: REAL := 41.0);
    PORT (TERMINAL a, b: ELECTRICAL;
          TERMINAL t  : THERMAL);
END electrothermal_actuator;
```

```
ARCHITECTURE behavoir OF electrothermal_actuator IS
    QUANTITY Lh : REAL := 220.0e-6;
    QUANTITY Lg : REAL := 2.0e-6;
    QUANTITY Lc : REAL := 182.0e-6;
    QUANTITY Lch : REAL := 2.0e-6;
    QUANTITY Lf : REAL := 38.0e-6;
    QUANTITY wh : REAL := 2.0e-6;
    QUANTITY wc : REAL := 16.0e-6;
    QUANTITY wf : REAL := 2.0e-6;
    QUANTITY hh : REAL := 2.0e-6;
    QUANTITY hc : REAL := 2.0e-6;
    QUANTITY hf : REAL := 2.0e-6;
    QUANTITY Vab ACROSS Ia THROUGH a TC b;
    QUANTITY Temp ACROSS Thermal_power THROUGH t TC thermal_ground;
    QUANTITY Temp0 : REAL := 25.0;
    QUANTITY R0, Ra, disp, x, h: REAL;
    QUANTITY Temp_max: REAL;
BEGIN
    -- electrothermal model part
    Temp == (Vab*Vab)/(2*Lh*Lh*rho*Kp)*(Lh*x-x*x)+Temp0;
    Temp_max == (Vab*Vab)/(8*rho*Kp)+Temp0;
    -- mechanical model part
    h == (Lch/Lh)*(wh+wc+Lg)+(Lc-Lch)/Lh*(wh+wc)+(Lf/Lh)/(wf+wc);
    disp == -alfa/(2*h)*(Temp-Temp0)*x*x;
    -- electrical model part
    R0 == rho*(Lh/(wh*hh)+Lc/(wc*hc)+Lf/(wf*hf));
    Ra == R0*(1+ksi*(Temp-Temp0));
    Ia == Vab/Ra;
    Thermal_power == Ia*Vab;
END behavior;
```

Fig. 3. Fragment of VHDL-AMS model of electro-thermal microactuator

V. SIMULATION RESULTS

The developed VHDL-AMS model allows to simulate the dependence of the flexure of hot (thin) and cold (wide) arms on their lengths and the applied voltage between the anchors of the microactuator, dependence of the current on the applied voltage and the arm length of the microactuator, temperature distribution along the arms, dependence of the power consumption on the applied voltage. Simulation results are graphically presented in Fig. 4 – 11.

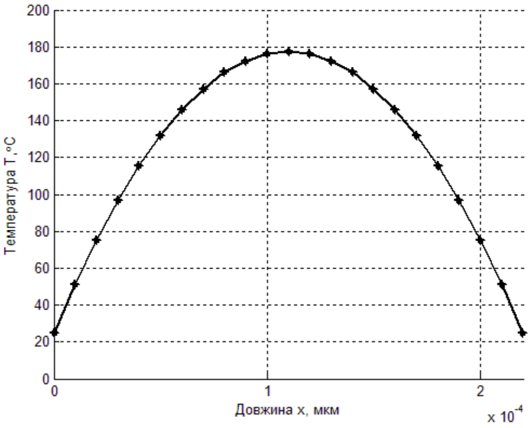

Fig. 4. Temperature distribution along the microactuator arm

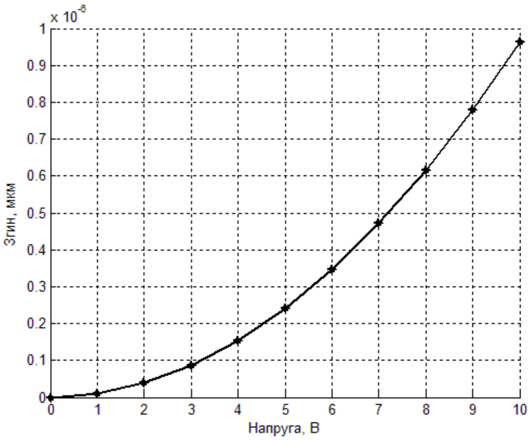

Fig. 6. Dependence of the microactuator bending on the voltage applied between its anchors

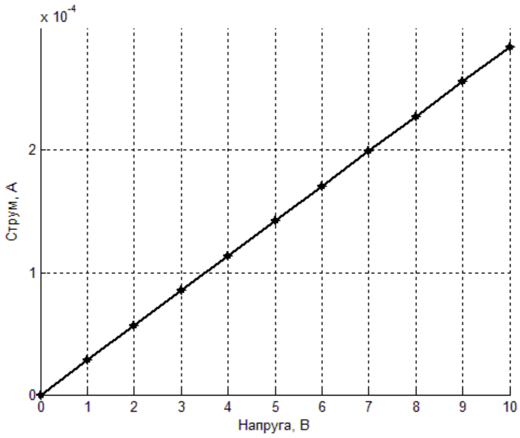

Fig. 7. Dependence of the current on the applied voltage between the microactuator anchors

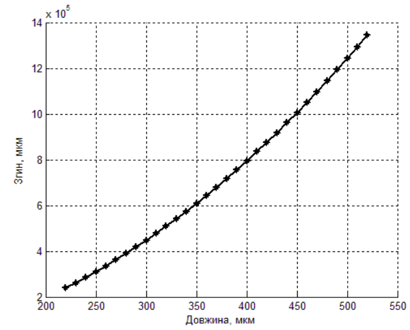

Fig. 8. Dependence of the actuator bending on the arm length

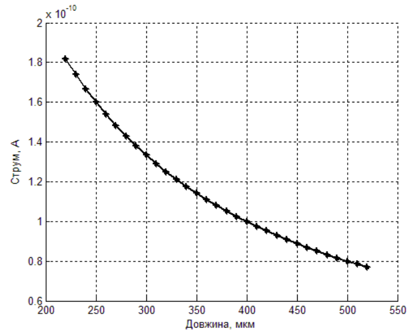

Fig. 9. Dependence of the current on the actuator arm length

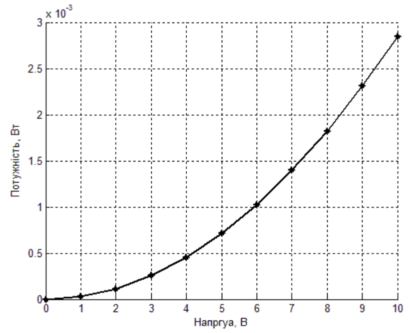

Fig. 10. Dependence of the power consumption of the microactuator on the applied voltage

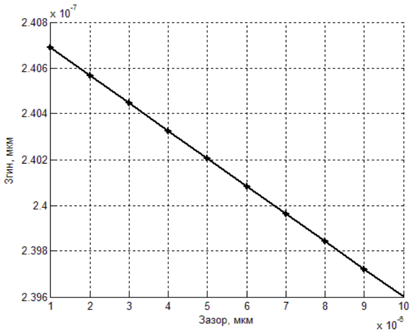

Fig. 11. Dependence of the microactuator bending on the gap between its thin (hot) and wide (cold) arms

REFERENCES

[1] Dong Yan. Mechanical Design and Modeling of MEMS Thermal Actuators for RF Applicationsю / Dong Yan // Thesis.– University of Waterloo, Ontario, 2002 – pp. 93.

[2] Shannon Zelinski. Design of Vertical-Lateral Thermal Actuators for MEMS / Shannon Zelinski. // Department of Electrical Engineering and Computer Sciences. – University of California, Berkeley, 2001 – pp. 4.

[3] Amarendra Atre. Design of a Micromachined Electro-Thermal Beam Flexure Polysilicon Actuator / Amarendra Atre // NSTI-Nanotech – 2005 – pp.

[4] Ang Beng Seng. Design and Analysis of Thermal Microactuator / Ang Beng Seng // European Journal of Scientific Research – 2009 – p. 281-292.

[5] Pavel V. VHDL-AMS based modeling and simulation of mixed-technology Microsystems / Pavel V. Nikitin, C.-J. Richard Shi // Tutorial – 2005 – p. 261.

[6] Qing Ji. First Order Modeling of Thermal Actuators in SUGAR / Qing Ji, Karen L. Scott // University of California – 2005 – p. 4.

[7] Krassimir Hristov Denishev. Thermal Microactuator / Krassimir Hristov Denishev, Eleonora Zhivkova Krumova // ELECTRONICS'2005 – 2005 – pp. 6.

[8] Michael S. Final Report: Compliant Thermo-Mechanical MEMS Actuators LDRD #52553 / Michael S. Baker, Richard A. Plass, Thomas J. Headley, Jeremy A. Walvaren // Sandia National Laboratories –2004.

[9] hAMSter Software for VHDL-AMS Simulations, http://www.theoinf.tu-ilmenau.de/~twangl/VHDL-AMS_online_en/Home.html.

[10] Peter J. Ashenden EDA CONSULTANT, ASHENDEN DESIGNS PTY. LTD., "VHDL Tutorial", Elsevier Science 2004 – pp. 84.

[11] Standard VHDL Analog and Mixed-Signal Extensions - Packages for Multiple Energy Domain Support – 2003 – pp. 21.

[12] VHDL 1076.1: Analog Extensions to VHDL, Ernst Christen, Analogy Inc. – April 1997 – pp. 9 .

VI. CONCLUSIONS

VHDL-AMS model of electro-thermal microactuator for computer-aided design is developed using hAMSter software. The developed model allows to simulate the dependence of the flexure of hot (thin) and cold (wide) arms on their lengths and the applied voltage between the anchors of the microactuator, dependence of the current on the applied voltage and the arm length of the microactuator, temperature distribution along the arms, dependence of the power consumption on the applied voltage, and also to perform the behavioral analysis of this device at the functional design level.

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